Exact Solutions for Hydromagnetic Flow of a Particulate Suspension

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Nomenclature

B	= magnetic induction field
b	= fluid-phase body force
b_p	= particle-phase body force
e_x, e_y	= unit vectors in the x and y directions, respectively
f	= interphase force
$\underline{I}$	= unit tensor
N	= interphase momentum transfer coefficient
P	= fluid pressure
V	= fluid-phase velocity
V_p	= particle-phase velocity
μ	= fluid dynamic velocity
ρ	= fluid density
ρ_p	= particle-phase in-suspension density (mass of particulate material per unit volume suspension)
ρ_{p_∞}	= freestream particle-phase density
$\underline{\sigma}$	= fluid-phase stress tensor
$\underline{\sigma}_p$	= particle phase stress tensor
σ_1	= electrical conductivity
ν	= fluid kinematic viscosity
∇	= gradient operator

Superscript

T	= transpose of a second-order tensor

Introduction

M ANY papers concerning single- and two-phase flows in the presence of a magentic field have been published.[1-11] Chamkha and Peddieson[12] reported exact solutions for the asymptotic suction profile for a two-phase suspension in the absence of a magnetic field.

As far as the author is aware, the problem of a particulate suspension in an electrically conducting fluid in the presence of a magnetic field past an infinite porous flat plate has not been solved. In the present Note, the author intends to extend the work of Chamkha and Peddieson[12] to include a uniform transverse magnetic field acting along the y axis. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. Since no external electric field is applied, and the effect of polarization of ionized fluid is negligible, it can be assumed that the electric field is negligible. Numerical computations of the exact solutions are performed, and some of the results are presented graphically to show special features of two-phase flows in the presence of a uniform transverse magnetic field.

Governing Equations

Consider the two-dimensional, steady, laminar, hydromagnetic flow that takes place in a half-space bounded by an infinite porous flat plate. Let the flow be a uniform stream parallel to the x,y plane with the plate being coincident with the plane $y = 0$. Far from the plate, both phases are in equilibrium with speed V_∞ in the x direction. Uniform suction with speed v_o is imposed on the fluid phase. The fluid is assumed to

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be incompressible, the particulate phase is assumed to be pressureless, and the volume fraction of suspended particles is assumed to be small.

The governing equations are based on the balances of mass

$$\nabla \cdot V = 0 \tag{1}$$

$$\nabla \cdot (\rho_p V_p) = 0 \tag{2}$$

(for the fluid and particulate phases, respectively) and the balances of linear momentum

$$\rho(V \cdot \nabla V) = \nabla \cdot \underline{\sigma} - f + b \tag{3}$$

$$\rho_p(V_p \cdot \nabla V_p) = \nabla \cdot \underline{\sigma}_p + f + b_p \tag{4}$$

(for the fluid and particulate phases, respectively) and the constitutive equations

$$\underline{\sigma} = -P\underline{I} + \mu(\nabla V + \nabla V^T)$$

$$\underline{\sigma}_p = 0$$

$$f = N\rho_p(V - V_p) \tag{5}$$

$$b = \sigma_1/\rho(V \times B) \times B$$

$$b_p = 0$$

Equations (1-5) constitute a generalization of the original dusty-gas model discussed by Marble[13] to include external body forces such as magnetic induction force.

The nondimensional governing equations can be obtained by substituting

$$x = \nu\xi/V_\infty, \quad y = \nu\eta/V_\infty, \quad V = e_x V_\infty F(\eta) - e_y v_0$$

$$V_p = e_x V_\infty F_p(\eta) + e_y v_\infty G_p(\eta) \tag{6}$$

$$B = B_o e_y, \quad P = \rho V_\infty^2 [L(\xi) + H(\eta)], \quad \rho_p = \rho_{p\infty} Q_p(\eta)$$

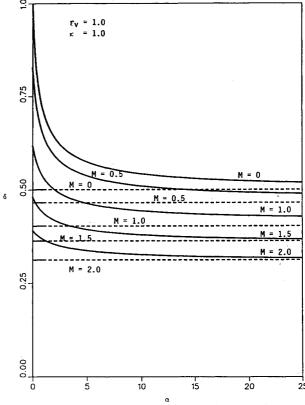Fig. 1 Fluid-phase displacement thickness vs α.

into Eqs. (1–5) and combining and rearranging to yield

$$r_v Q_p'/Q_p^2 - \alpha(Q_p - 1) = 0 \qquad (7)$$

$$F'' + r_v F' + \kappa\alpha Q_p(F_p - F) - M^2(F - 1) = 0 \qquad (8)$$

$$G_p F_p' + \alpha(F_p - F) = 0 \qquad (9)$$

$$G_p = -r_v/Q_p, \qquad H' = \kappa\alpha r_v (Q_p - 1) \qquad (10)$$

where a prime denotes an ordinary differentiation with respect to η, and $r_v = v_0/V_\infty$, $\alpha = N\nu/V_\infty^2$, $\kappa = \rho_{p\infty}/\rho$, and $M = \sqrt{\sigma_1/\mu}$ $B_o\nu/V_\infty$ are the suction parameter, the inverse Stokes number, the particle loading, and the Hartmann number, respectively. In the present work, μ, N, and B_o will all be assumed constants.

It should be mentioned that Eqs. (7–10) apply exactly for an infinite plate whereas they apply far from the leading edge or for a large amount of suction for a semi-infinite plate so that the nonlinear convective acceleration terms can be neglected. The boundary conditions are

$$F(0) = 0, \qquad H(0) = 0 \qquad (11)$$

$$\lim_{\eta \to \infty} F(\eta) = 1, \quad \lim_{\eta \to \infty} F_p(\eta) = 1, \quad \lim_{\eta \to \infty} Q_p(\eta) = 1 \qquad (12)$$

Regardless of the values of r_v and α, $Q_p = 1$ satisfies Eq. (7). This implies that $G_p = -r_v$ and $H = 0$ $(L = -M^2\xi)$. The results obtained in the present Note are based on these solutions.

The displacement thickness for both the fluid and particle phases and the fluid-phase skin-friction coefficient are defined, respectively, as

$$\delta = \int_0^\infty (1 - F)\, d\eta, \quad \delta_p = \int_0^\infty (1 - F_p)\, d\eta, \quad C_f = F'(0) \qquad (13)$$

Results

Equations (8) and (9) are linear equations with constant coefficients, which can be solved for F and F_p, respectively,
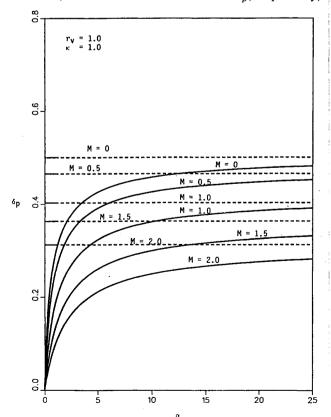

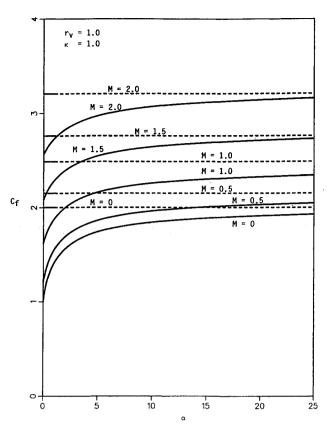Fig. 2 Particle-phase displacement thickness vs α.

Fig. 3 Fluid-phase skin friction coefficient vs α.

subject to Eqs. (12) by the usual method for solving such equations. This linearity property is due to the assumption made earlier that μ, N, and B_o are constants. It can be shown that the appropriate solutions for F and F_p, respectively, are

$$F = 1 - \exp(-\lambda_1\eta), \qquad F_p = 1 - \alpha/(\alpha + r_v\lambda_1)\exp(-\lambda_1\eta) \qquad (14)$$

where λ_1 is the absolute value of the only negative root of the cubic equation

$$r_v\lambda^3 + (r_v^2 - \alpha)\lambda^2 - r_v[\alpha(1 + \kappa) + M^2]\lambda + M^2\alpha = 0 \qquad (15)$$

It should be pointed out that the acceptable physical solution for Eq. (15) is to have one negative and two positive real roots. (Complex roots will produce oscillatory behavior, three positive roots will cause the constants of integration to be overdetermined, and more than one negative root will cause the constants of integration to be underdetermined.) It could not be analytically proven (using the equation associated with the cubic formula) that there must be one negative and two positive real roots. In all of the specific numerical examples considered, however, one negative and two positive real roots were found.

The appropriate expressions for the displacement thicknesses for both the fluid and particle phases and the skin-friction coefficient for the fluid phase can be written, respectively, as

$$\delta = 1/\lambda_1, \quad \delta_p = \alpha/[\lambda_1(\alpha + r_v\lambda_1)], \quad C_f = \lambda_1 \qquad (16)$$

Some limiting solutions for various special cases can be obtained. This will be done next.

When $\alpha \to 0$ (frozen flow), both the fluid and particulate phases move independently. Under these circumstances, the solutions already given reduce to

$$F = 1 - \exp(-\lambda_1^*\eta) \qquad (17a)$$

$$F_p = 1 \qquad (17b)$$

$$\delta = 1/\lambda_1^*, \qquad \delta_p = 0, \qquad C_f = \lambda_1^* \qquad (18)$$

where

$$\lambda_1^* = K + \sqrt{K^2 + M^2}; \qquad K = r_v/2$$

Equation (17a) is the single-phase asymptotic suction profile in the presence of a uniform transverse magnetic field. This solution is consistent with that reported by Singh.[7]

When $\alpha \rightarrow \infty$ (equilibrium flow), both phases move together with the same velocity. For this case, Eqs. (14) and (16) can be expressed as

$$F = 1 - \exp(-\lambda_1^{**}\eta) \qquad (19a)$$

$$F_p = 1 \qquad (19b)$$

$$\delta = \delta_p = 1/\lambda_1^{**}, \qquad C_f = \lambda_1^{**} \qquad (20)$$

where

$$\lambda_1^{**} = K(1 + \kappa) + \sqrt{K^2(1+\kappa)^2 + M^2} \qquad (21)$$

Equation (19a) can be thought of as the asymptotic suction profile in the presence of a uniform transverse magnetic field for a single fluid having density $(1 + \kappa)\rho = \rho + \rho_p$.

When $\kappa \rightarrow 0$ (dilute limit), the behavior of the fluid phase becomes independent of the presence of particles. As mentioned by Soo,[14] this is the simplest nontrivial case of dispersed two-phase flow. Upon equating $\kappa = 0$ in Eqs. (14) and (16), the corresponding solutions assume the forms

$$F = 1 - \exp(-\lambda_1^*\eta), \qquad F_p = 1 - \alpha/(\alpha + r_v\lambda_1^*)\exp(-\lambda_1^*\eta) \qquad (22)$$

$$\delta = 1/\lambda_1^*, \qquad \delta_p = \alpha/[\lambda_1^*(\alpha + r_v\lambda_1^*)], \qquad C_f = \lambda_1^* \qquad (23)$$

It should be mentioned that all of the solutions obtained in the present Note reduce to the results reported by Chamkha and Peddieson[12] if M is equated to zero.

The main purpose of this Note is to show the influence of the magnetic field on the flow properties. This can be seen from Figs. 1–3.

Figures 1–3 present the fluid-phase displacement thickness δ, the particle-phase displacement thickness δ_p, and the fluid-phase skin-friction coefficient C_f vs the inverse Stokes number α for various values of the Hartmann number M, respectively. It can be seen easily from these figures that increases in the Hartmann number M cause increases in the skin-friction coefficient C_f and decreases in the displacement thicknesses for both the fluid and particulate phases δ and δ_p, respectively. The dotted lines correspond to the equilibrium limit.

Conclusions

In the present Note, closed-form solutions for the hydromagnetic flow of a particulate suspension past an infinite porous flat plate were obtained. Some limiting solutions were developed for special cases. Graphical results of the exact solutions were presented and used to show the influence of the presence of a magnetic field on the flow properties. It was concluded that the skin-friction coefficient for the fluid phase increases whereas the displacement thicknesses for both the fluid and particulate phases decrease as the strength of the magnetic field increases.

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Vibration Mode Shape Control by Prestressing

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Introduction

IN many applications we have points on a structure that need to be protected from excessive vibration. A common example is that of a car where the designer attempts to obtain low vibration amplitudes under the driver's seat. In the case of large space structures, the position of sensitive instruments is expected to dictate the need for such vibration protection at selected points. In the present Note we are concerned with excitation in a narrow frequency band, so that only a small number of vibration modes contribute to the intensity of the forced response. In many cases it is not feasible to move the natural vibration frequencies of the structure far enough from this excitation band, so that we need to consider instead the expedient of reshaping the vibration modes to reduce the amplitude of vibration at the critical point or points.